

COMPOSITE HOUGH TRANSFORM FOR MULTITARGET MULTISENSOR TRACKING

10

BACKGROUND OF THE INVENTION

Field of the Invention

This invention pertains generally to target tracking of underwater objects and more particularly to the tracking of multiple targets using distributed acoustic linear sensor arrays utilizing spatial matched filtering techniques that can combine different image features from different image data in a coherent manner.

Description of the Related Art

20 The delay curve Hough transform (DCHT) was developed based on the paradigm of track-before-detect and applied to image data collected from a single sensor array for target tracking and track parameter estimations. Two different kinds of target tracking ambiguities are associated with a single sensor array using DCHT. See, Stevens et al., APPLICATION OF THE HOUGH TRANSFORM TO ACOUSTIC BROADBAND CORRELATIONS FOR PASSIVE
25 DETECTION AND LOCATION; NRL Mem. Rpt. NRL/MR/5580-92-7182, January 1993. One of them is the mirror track ambiguity and the other is the delay curve parameter ambiguity. It is well known that multiple sensor systems are required to address these problems.

The DCHT is closely related to techniques of classical passive dual-channel localization.

5 It uses a pattern recognition technique, the Hough transform, to operate on a set of continuous snapshots of cross-correlation, i.e., broadband correlation, between the two channels and perform target detection and target track parameter estimation. A broadband signal source with a constant speed, moving along a linear track, can generate a correlation trace in a broadband correlogram. This correlation trace can be described by an analytic equation, called the delay 10 curve, determined by the track direction, the closest point of approach (CPA) time t_0 , and the ratio between the closest point of approach horizontal distance D and speed v. Taking advantage of this delay curve, the DCHT hypothesizes a group of potential delay curves; accumulates evidence for each delay curve by summing (integrating) the pixel values along each hypothesized delay curve; stores accumulated pixel values in the parameter space (also known as 15 the Hough Space); and thresholds the accumulated pixel values to detect the delay curves. Since the detection is done in the parameter space, the parameters of the image feature are determined by the location of the peak.

When using multiple sensor systems, multiple targets generate multiple traces on different recordings produced from different sensor systems. The difficulties encountered using 20 multiple sensors are well-known issues of data association, such as measure-to-measure association -- how the target traces are formed and how they relate to the target's movement--; measurement-to-track association -- which trace should be followed when target traces cross each other--; and track-to-track association -- which target traces in different recording are associated with the same target. Different tracks with the same CPA time t_0 , track direction θ , and v/D ratio 25 but different speeds v and CPA horizontal distances D can generate the same delay curve, and

Inventor: Shyu
Serial No.

PATENT APPLICATION
Navy Case No. 79,739

5 hence, the DCHT cannot differentiate among such tracks. Also, given a linear sensor array, for any track, a mirror track exists that is the mirror reflection of the original track with respect to the linear sensor array. Due to the inherent geometric symmetry in a linear array, these two tracks produce the same delay curve in the broadband correlogram; hence, the DCHT cannot distinguish these two tracks.

10 The DCHT as a constant-speed signal source moves along a linear track in the neighborhood of a dual-sensor system, the time delay between the signal arriving at the two sensors follows an analytic equation called the delay curve. The two sensor system can consist of two omnidirectional hydrophones, two halves of a split array, or two individual arrays. Assuming a plane wavefront arrival at two sensors at the same depth, Sevens et al., *supra*,

15 showed that the equation of the delay curve is given by:

$$\tau(t) = \tau_{\max} \frac{\left(\frac{v}{D}(t - t_o)\right) \cos \theta - \sin \theta}{\sqrt{1 + \left(\frac{v}{D}(t - t_o)\right)^2 + \left(\frac{h}{D}\right)^2}} \quad (1)$$

where $\tau(t)$ is relative time delay between the two received signals at time t ; τ_{\max} is the length of the baseline divided by the signal propagation speed; t_o is CPA time; D is horizontal distance at CPA from the signal source to the midpoint of the sensor pair; h is depth difference between the signal source and the sensors; v is the signal source speed; θ is the track direction defined by the

Inventor: Shyu
Serial No.

PATENT APPLICATION
Navy Case No. 79,739

5 right-turn rule convention; and t is observation time.

From the analytic equation for the delay curve, Eq. (1), the delay curve Hough transformation can be defined mathematically as

$$f\left(\frac{v}{D}, \theta, t_o, \frac{h}{D}\right) = \frac{1}{N} \iint F(x, y) \delta(\tau(x, y)) dx dy \quad (2)$$

10 where $f(v/D, \theta, t_o, h/D)$ is the output of the DCHT; $F(x, y)$ is pixel value in the correlogram at location (x, y) ; $\delta()$ is Dirac-delta function restricting the integration to the delay curve; N is the total number of pixels in the integration; x is horizontal offset in correlogram, and y is vertical offset in correlogram.

15 The target track is a function of five track parameters: v , θ , h , D , and t_o . The delay curve is a function of the four delay parameters: θ , t_o , and the ratios v/D and h/D . The DCHT is defined over the set of delay curves parameters, and thus the parameter space of the DCHT has only four dimensions: θ , t_o , v/D , and h/D . Each dimension is independently sampled over an appropriate range for the tracks of interest. For simplification, the depth factor, h/D , is assumed to be a constant.

20 As previously discussed, two target-tracking problems are associated with a single linear array. The first problem is the so-called delay curve parameter ambiguity different target tracks with different track parameters, v , D , θ , t_o , but having the same delay curve parameters, v/D , θ , t_o , will produce the same delay curve in the correlogram, hence are indistinguishable in the correlogram. The second problem is the mirror effect. For every individual target track there

Inventor: Shyu
Serial No.

PATENT APPLICATION
Navy Case No. 79,739

5 exists a mirror track; because of geometrical symmetry, the target track and its mirror track are indistinguishable in the correlogram since they always have the same time delay to the two sensors.

Figures 1a and 1b shows an example of the delay curve parameter ambiguity. Figure 1a shows the target-sensor geometry, and Figure 1b shows its corresponding delay curve. Even though the target track 11 is a straight line, the delay curve 13 in the correlogram veers sharply. Two parallel dashed lines 15 and 17 representing two different target tracks are also shown in Figure 1a; these dashed track lines 15 and 17 indicate different target tracks but with the same delay curve parameters; same direction, same CPA lines, the same v/D ratio. To have the same v/D ratio, a target track with a longer CPA distance must have a higher target speed and vice versa. Because their delay curve parameters are identical, these different tracks generate exactly the same delay curve in the correlogram. Without additional information, the DCHT cannot differentiate between these tracks.

Figure 2 shows the mirror effect; two target tracks are shown. One is the reference track 19; the other one is the mirror reflection 21 of the reference track 19. Although the mirror track 21 is geometrically distinct from the reference track 19, the linear array cannot differentiate between the two because the correlation trace generated by the mirror track 21 is exactly the same as that generated by the reference track 19. It is useful to note that a track and its mirror track will have opposite signs (positive and negative) with respect to the right-turn rule for determining track direction, discussed above,

25 From the foregoing discussion, both the target track 11 and delay curve 13 are

Inventor: Shyu
Serial No.

PATENT APPLICATION
Navy Case No. 79,739

5 characterized by the track parameters v , D , θ , and t_o . Using the DCHT, the delay curve 13 can be detected from the broadband correlogram, and the corresponding parameters can be extracted. The target track 11 can be reconstructed, but with some degree of ambiguity. The delay curve 13 equation, Eq. (1), can be viewed as a mapping between the target track 11 space and the delay curve 13 space. This mapping, however, is not a one-on-one mapping. Instead, because of the 10 delay curve 13 parameter ambiguity and the mirror effect, it may be a many-to-one mapping. The key problem is how to resolve this many-to-one mapping to recover the actual target track from the correlation trace in the correlograms. The answer lies in using multiple arrays.

SUMMARY OF THE INVENTION

15 The object of this invention is to provide a filtering technique that can combine different image features from different image data in a coherent manner when tracking multiple targets using distributed linear acoustic linear sensor arrays.

Another object of this invention is to provide for multitarget-multisensor tracking utilizing real data sets from a shallow-water environment.

20 Another object of this invention is to provide a device that does not require an initial guess of the number and state of the targets, and can be applied to multitarget-multisensor tracking without an initial guess, unlike conventional Kalman-filter based techniques.

Another object of this invention is to exploit the geometric properties of multiple arrays and perform data fusion for multiple sensor arrays, resolving ambiguity due to the mirror effect 25 and the delay curve parameter ambiguity associated with a single array, and allowing the

5 reconstruction of target tracks.

Another object of this invention is to enable one to perform noncoherent integration from sweep to sweep, from delay-time-cell to delay-time-cell, and from array to array while providing a processing gain over a single array.

10 Another object of this invention is to provide for increased processing gain and to facilitate sensor systems configuration and management for multiple array systems.

Another object of this invention is to suppress sidelobes and obtain a sharper peak and improve the accuracy of the parameter estimation with sensor systems.

Another object of this invention is to provide a device that reduces a large number of calculations to search over the full space.

15 This and other objectives are achieved by the use of a Composite Hough Transform (CHT) in a sensor system. The CHT operates on two or more corresponding broadband correlograms produced at two neighboring dual-channel sensor systems. A broadband signal source can generate one correlation trace on each of the correlograms. Since these correlation traces are produced by the same signal source, they are constrained by a set of geometric relationships. By fully exploiting this set of constraints, the CHT fuses sensor data from multiple 20 dual-channel sensor systems for target detection and track parameter estimation. The dual-channel system is a split-array system; a linear array split into two subarrays with each subarray corresponding to one of the channels. The CHT operates with two neighboring linear subarrays. One of the sensor arrays is chosen as the primary array; it is used to originate the different track 25 hypotheses for the signal source. Its corresponding broadband correlogram is referred to as the

5 primary correlogram. The other sensor array is the secondary array, and the corresponding broadband correlogram is referred to as the secondary correlogram. The CHT exploits the geometric relationships between the primary and secondary array.

BRIEF DESCRIPTION OF THE DRAWINGS

10 **Figure 1a** shows a target-sensor geometry.

Figure 1b shows a corresponding delay curve for the target-sensor geometry shown in
Figure 1a.

Figure 2 shows the mirror effect.

Figure 3a shows a broadband correlogram array A of two neighboring sensor arrays.

15 **Figure 3b** shows a broadband correlogram array B of two neighboring sensor arrays.

Figure 4a shows a sensor-target geometry for use with a composite Hough transform.

Figure 4b shows a system diagram for a composite Hough transform.

Figure 5 shows the relative geometry between the signal source and the two sensors.

Figure 6a shows the right-turn-rule convention for the target direction for $0^\circ < \theta < 90^\circ$.

20 **Figure 6b** shows the right-turn-rule convention for the target direction for $90^\circ < \theta < 180^\circ$.

Figure 6c shows the right-turn-rule convention for the target direction for $180^\circ < \theta < 270^\circ$.

Figure 6d shows the right-turn-rule convention for the target direction for $270^\circ < \theta < 360^\circ$.

5 **Figure 7a** shows a target -sensor geometry for a scenario of delay curve parameter ambiguity and the mirror effect.

Figure 7b shows a delay curve at array A1 for a scenario of delay curve parameter ambiguity and the mirror effect.

10 **Figure 7c** shows the reference track delay curves observed by array A2 for a scenario of delay curve parameter ambiguity and the mirror effect.

Figure 7d shows the mirror track delay curves observed by array A2 for a scenario of delay curve parameter ambiguity and the mirror effect.

Figure 8 shows a generic target-sensor geometry.

15 **Figure 9a** shows the corresponding delay curve for **Figure 6** observed by the primary array.

Figure 9b shows the delay curve for **Figure 6** observed by the secondary array.

Figure 10 shows a data flow diagram of the Composite Hough Transform.

Figure 11a shows target-sensor geometry for basic behavior of Composite Hough Transform.

20 **Figure 11b** shows a simulated correlogram from a primary array A₁.

Figure 11c shows a simulated correlogram from a secondary array A₂.

Figure 12a shows the Reference Track Composite Hough Space produced by ACHT for v=9.0 kt and D₁ = 1.0 nmi.

25 **Figure 12b** shows the Mirror Track Composite Hough Space produced by ACHT for v=9.0 kt and D₁ = 1.0 nmi.

5 **Figure 13a** shows the target sensor geometry showing how the Composite Hough Transform resolves the mirror effect

Figure 13b shows the simulated primary correlogram from array A_1 with correlation trace due to the target track.

10 **Figure 13c** shows the simulated secondary correlogram from array A_2 with correlation trace due to the target track.

Figure 13d shows the simulated secondary correlogram from array A_2 with correlation trace due to the mirror track.

Figure 14a shows the Composite Hough Space produced by the ACHT, using **Figure 13a** as the primary correlogram.

15 **Figure 14b** shows the Composite Hough Space produced by the MCHT, using **Figure 13d** as the secondary correlogram.

Figure 15a shows the target-sensor geometry for a scenario of delay curve parameter ambiguity.

Figure 15b shows the primary correlogram, with only one correlation trace.

20 **Figure 15c** shows the secondary correlogram, with two correlation traces produced by the two tracks.

Figure 16a shows the first detected correlation trace as a result of a first layer onion-peeling on the primary correlation.

25 **Figure 16b** shows the first detected correlation trace as a result of a first layer onion-peeling on the secondary correlation.

5 **Figure 16c** shows the second detected correlation trace as a result of a second layer onion-peeling on the primary correlation.

Figure 16d shows the second detected correlation trace as a result of a second layer onion-peeling on the secondary correlation.

10 **Figure 17a** shows the slice of RTCHS produced by the ACHT for a speed of 4.5 kts and D_1 of 1.0 nmi. **Figures 17c and 17d** shows the slice of RTCHS and MTCHS for a speed of 9.0 kts and D_1 of 2.0 nmi.

Figure 17b shows the slice of and MTCHS produced by the ACHT for a speed of 4.5 kts and D_1 of 1.0 nmi.

Figures 17c shows the slice of RTCHS for a speed of 9.0 kts and D_1 of 2.0 nmi.

15 **Figure 17d** shows the slice of MTCHS for a speed of 9.0 kts and D_1 of 2.0 nmi.

Figure 18a shows the output Reference Track Composite Hough Space produced by the ACHT;

Figure 18b shows the output Mirror Track Composite Hough Space produced by the ACHT

20 **Figure 18c** shows the output Reference Track Composite Hough Space produced by the MCHT

Figure 18d shows the output Mirror Track Composite Hough Space produced by the correlograms shown in **Figure 20**.

Figure 22c shows RTCHS at the first onion-peeling produced by MCHT.

25 **Figure 22d** shows MTCHS at the first onion-peeling produced by MCHT.

5 **Figure 22e** shows RTCHS at the second onion-peeling produced by MCHT.

Figure 22f shows MTCHS at the second onion-peeling produced by MCHT.

Figure 23a shows the broadband correlogram from the primary array as a result of the onion-peeled Additive Composite Hough Transform.

Figure 23b shows the broadband correlogram from the secondary array as a result of the onion-peeled Additive Composite Hough Transform.

10 **Figure 24** shows the sensor geometry and the reconstructed target tracks

Figure 25a shows the ACHT's output Composite Hough Space for the real data for RTCHS at $v = 9.0$ kts, $T1 = 2.99$ nmi.

Figure 25b shows the MCHT's output Composite Hough Space for the real data for RTCHS at $v = 9.0$ kts, $T1 = 2.99$ nmi.

15 **Figure 25c** shows the RCHT's output Composite Hough Space for the real data for RTCHS at $v = 8.5$ kts, $T1 = 12.84$ nmi.

Figure 25d shows the MCHT's output Composite Hough Space for the real data for RTCHS at $v = 8.5$ kts, $T1 = 12.84$ nmi.

20 **Figure 25e** shows the RCHT's output Composite Hough Space for the real data for RTCHS at $v = 9.0$ kts, $T1 = 12.84$ nmi.

Figure 25f shows the MCHT's output Composite Hough Space for the real data for RTCHS at $v = 9.0$ kts, $T1 = 12.84$ nmi.

Figure 25g shows the RCHT's output Composite Hough Space for the real data for RTCHS at $v = 8.5$ kts, $T1 = 12.84$ nmi.

5 **Figure 25h** shows the MCHT's output Composite Hough Space for the real data for
RTCHS at $v = 8.5$ kts, $T1 = 12.84$ nmi.

Figure 25i shows the RCHT's output Composite Hough Space for the real data for
RTCHS at $v = 7.0$ kts, $T1 = 8.92$ nmi.

10 **Figure 25j** shows the MCHT's output Composite Hough Space for the real data for
RTCHS at $v = 7.0$ kts, $T1 = 8.92$ nmi.

Figure 26 shows a basic flow chart for an additive composite Hough transform.

Figure 27 shows a basic flow chart for an additive composite Hough transform.

Figure 28 shows a basic flow chart for an additive composite Hough transform.

Figure 29 shows a basic flow chart for an additive composite Hough transform.

15 **Figure 30** shows a basic flow chart for an additive composite Hough transform.

Figure 31 shows a basic flow chart for an additive composite Hough transform.

Figure 32 shows a basic flow chart for an additive composite Hough transform.

DETAILED DESCRIPTION OF THE INVENTION

20 An extension of Delay Curve Hough Transform (DCHT) is the Composite Hough
Transform (CHT). Unlike the DCHT operating on a broadband correlogram produced by a single
dual-channel sensor system, the CHT operates on multiple corresponding broadband correlograms
produced at multiple neighboring dual-channel sensor systems. A broadband signal source can
generate one correlation trace on each of the correlograms. Since these correlation traces are
25 produced by the same signal source, they are constrained by a set of geometric relationships. By

5 fully exploiting this set of constraints, the CHT fuses sensor data from multiple dual-channel sensor systems for target detection and track parameter estimation. The dual-channel system can be a split-array system; a linear array split into two subarrays with each subarray corresponding to one of the channels or two neighboring linear subarrays. One of the sensor arrays is chosen as the primary array; it is used to originate the different track hypotheses for the signal source. Its
10 corresponding broadband correlogram is referred to as the primary correlogram. The other sensor array is the secondary array, and the corresponding broadband correlogram is referred to as the secondary correlogram.

The CHT exploits the geometric relationships between the primary and secondary array. Similar to the DCHT, the CHT hypothesizes a track, called the reference track, relative to the
15 primary array; calculates the associated delay curve in the primary correlogram; derives, based on the geometric constraints, the delay curve for the track on the secondary correlogram; accumulates evidence for the track by simultaneously summing (integrating) the pixel values along the appropriate delay curves in the primary and the secondary correlograms; stores the accumulated pixel values in the composite Hough space; and thresholds the accumulated pixel values to detect
20 the track.

Since the detection is done in the parameter space, the parameters of the delay curves are determined by the location of the peak. By using two correlograms, the CHT resolves the target tracking ambiguities associated with a single linear sensor array and provides another 3 dB processing gain over the DCHT.

25 **Figures 3a and 3b** shows two typical real data broadband correlograms from two

5 neighboring arrays; a primary array, **Figure 3a**, and a secondary array, **Figure 3b**. The vertical axis is the observation time. The horizontal axis is the relative time delay, time difference of arrival (TDOA), of a signal at two sensors. Five different correlation traces are shown in each of the correlograms; each correlation trace corresponds to a target. In **Figure 3a**, target 1 **12** has a high change rate of bearing and moves from maximum TDOA to minimum TDOA; targets 2 **14**
10 and 4 **16** move from minimum TDOA towards zero TDOA; target 3 **18** maintains the maximum TDOA for most of the observation period with an apparent course change near the bottom of the figure; and target 5 **22** appears only in the beginning of the observation period.. In this case the secondary correlogram **20**, is almost the mirror reflection of the primary correlogram **10**, with the exception that the course change for target 3 **18** is more significant. This behavior indicates an
15 apparent one-to-one correspondence between the correlation traces on the two broadband correlograms. In general, the primary **10** and secondary **20** correlograms do not appear as near-mirror reflection of each other because of differences in array orientation and target tracks.

Using two different broadband correlograms to do multiple target tracking involves data association. Particularly, the three levels of data association previously discussed in relation to
20 Kalman-filter based techniques to be considered. These types of issues are common to the problem of multitarget-multisensor tracking. The first treatment of this type of problem was discussed in Singer et al., COMPUTER CONTROL OF MULTIPLE SITE TRACK CORRELATION; *Automatica*, Vol. 7, pp. 455-464, 1971. They assumed that a Kalman filter estimated the target's state, such as the target's position and speed. Assuming two sites, each with
25 its own sensor and data processor (Kalman filter), the difference of the two estimates was tested

5 for the hypothesis that the underlying states were the same.

Conventional multitarget-multisensor tracking techniques dissect the target-tracking problem into three stages: detection, track formation, and track maintenance. Typically, a predetermined threshold is used in the stage of detection; thresholding in this stage can potentially discard some important target information. Instead of discarding target signal power in the 10 detection stage, the CHT uses the geometric constraints relating the target and sensor geometry to a track-before-detection approach that integrates the signal power to accumulate a confidence level as part of the tracking process: thresholding is not done until the last stage.

To facilitate a top-level understanding, **Figure 4a** shows a sensor-target geometry for use with a system diagram for a composite Hough transform shown in **Figure 4b**. The acoustic 15 signals **24** and **26** collected by each corresponding array **28** and **32** are processed by a split-array beamformer and cross-correlated by a split-array cross-correlator **34** and **36** to generate a broadband correlogram **38** and **42**. The two broadband correlograms **38** and **42** are input to the CHT **44** for target detection and target track parameters estimation. The target tracks can be easily reconstructed from the track parameters; thus the CHT can be viewed as an inverse 20 mapping from the delay curves in the two correlograms **38** and **42** to a target track **46**. Sometimes this kind of two-sensor system is referred to as a dual-channel system, a more generic term indicating that the system can be operated with different types of sensor configuration. The two sensors are referred to as the left sensor and the right sensor. The line connecting the two sensors is the baseline, with the direction defined by the vector from the left sensor to the right sensor. 25 The direction of the baseline must be consistent with the direction used in producing the

5 broadband correlation. The center of the sensor system is the midpoint between the two sensors. A split array is a linear array split into two subarrays. Therefore, the two sensors of a dual-channel system are the subarrays of the array. **Figure 5** shows the relative geometry between the signal source and the two sensors. The signal source moves with a constant speed v along a linear track with direction θ and reaches its closest point of approach (CPA) to the center of the sensor
10 system at a distance D at time t_0 .

The track direction θ is the angle between the target track and the baseline of the sensor system **52**, measured counterclockwise, and defined by a right-turn rule convention. First, a CPA ray is drawn from the center of the two-sensor system **51** to the CPA point **53**. Second, move along the CPA ray **55**, and make a right turn at the intersection of the CPA ray and the target track
15 **48**. The track direction θ is positive if the target track **48** is headed in the direction of the right turn. Otherwise, the track direction θ is negative. **Figures 6a** through **6d** shows this convention. The two sensors **52** and **54** are indicated by two small ellipses; the center of the sensor system **58** is the origin; the direction of the sensor system **62** is to the east and the baseline **64** of the sensor system is the horizontal line. Arrows that point from the origin to the target CPA indicate the
20 CPA rays. The two target tracks **57** and **59** are shown in each of the four quadrants, **Figures 6a** through **6d**: one with a positive direction **57** and one with a negative direction **59**. The right-turn rule convention uniquely describes all possible target tracks.

A two-linear-sensor-array system is used to resolve the two target-tracking problems associated with a single linear array. **Figures 7a** through **7d** shows a sensor-target scenario
25 associated with the delay curve parameter ambiguity and the mirror effect. **Figure 7a** shows two

5 different source tracks (RT1 and RT2) **66** and **68**, respectively, with identical delay curve parameters (θ , t_0 , v/D) with respect to array A_1 **84** and their associated mirror tracks (MT1 and MT2) **88** and **92**, respectively, in the field of two linear arrays A_1 (primary array) **84** and A_2 (secondary array) **86**. The primary array **84** and the secondary array **86** have orientations of -33.61° and 151.28° respectively. The interarray distance is 4.867 km., with the center of array 10 A_1 **84** at $-1976.3, -729.45$ m and the center of array A_2 **86** at $2653.33, 773.28$ m. The observation period is 80 min. The reference tracks **66** and **68** have a direction of 243.6° relative to the primary array and reach their CPA 40 min. into the run. RT1 **66** has a speed of 4.5 knots and a CPA distance to A_1 **84** of 1.0 nmi. RT2 **68** has a speed of 9.0 knots and a CPA distance of 2.0 nmi. The two tracks, therefore, have the same v/D ratio of 4.5, and hence produce the same delay 15 curve in the primary correlogram. The difference between the reference tracks **66** and **68** and the mirror tracks **88** and **92** is the track direction relative to the primary array **84**. The reference tracks **66** and **68** have a direction of 243.6° relative to the primary array, and the mirror tracks **88** and **92** have a direction of -243.6° . The two mirror tracks **88** and **92** also produce the same delay curves 20 **72** in the primary correlogram. When viewed from the secondary array **86**, however, the two reference target tracks RT1 **66** and RT2 **68** and the two associated mirror tracks MT1 **88** and MT2 **92** have different delay curve parameters and produce different delay curves in the secondary correlogram. Therefore, by using the second array **86**, the mirror effect and the delay curve 25 parameter ambiguities are resolved. **Table 1** shows track parameters for the four tracks.

Figure 8 shows a generic target-sensor geometry. From this target-sensor geometry, the 25 geometric constraints between the primary array **94** and the secondary array **96** can be derived.

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TABLE 1

Track	v(kts)	$\theta_1(^{\circ})$	$D_1(\text{nmi})$	$t_{01}(\text{min})$	$\theta_2(^{\circ})$	$D_2(\text{nmi})$	$t_{02}(\text{min})$
RT1	4.5	243.6	1	40	58.7	0.453	14.29
RT2	9.0	243.6	2	40	58.7	1.453	27.15
MT1	4.5	-243.6	1	40	248.5	1.378	52.19
MT2	9.0	-243.6	2	40	248.5	0.378	45.60

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The primary array **A₁ 94**, and the secondary array **A₂ 96** have orientations of α_1 ° and α_2 °, respectively. The orientation of an array is defined as the direction (measured counterclockwise relative to east) of the ray from the phase center of one of the split arrays to the other. The baseline (i.e., the line segment connecting the centers of the two arrays) between the two linear arrays has length L and an orientation (i.e., the direction of the ray connecting the primary array to the secondary array) of β °. The CPA ray direction ω is defined as the angle difference between the CPA ray direction ω and the signal source track direction θ . The turn angle $\phi=\omega-\theta$ can only be either 90° or -90°. The target track satisfies the right turn rule when

$$\text{sign}(\sin(\phi))>0, \quad (3)$$

where $\text{sign}(X)$ is the sign function and has the value 1 if $X>=0$, and -1 if $X<0$. The target reaches its CPA to the primary array **94** at time t_{01} with distance D_1 ; and reaches its CPA to the secondary array **96** at time t_{02} with distance D_2 . **Table 2** summarizes the parameters used to describe the tracks with respect to these two arrays **94** and **96**.

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Figures 9a and **9b** shows the corresponding delay curves **98** and **102** observed by the primary array **94** and the secondary array **96**, respectively.

TABLE 2

	Symbol	Description
	v	target speed
10	θ	reference track direction relative to east
	θ_1	reference track direction relative to primary array
15	θ_2	reference track direction relative to secondary array
	α_1	orientation of the primary array
	α_2	orientation of the secondary array
	β	orientation of the array pair
20	δ	difference between reference track direction and orientation of the array pair
	D_1	CPA distance from center of primary array to reference track
	D_{2r}	CPA distance from center of secondary array to reference track
	t_{02}	CPA time of reference track with respect to secondary array
	ω_2	CPA ray direction for secondary array
	ϕ_{2r}	turn angle sat secondary array for reference track ($\omega_2 - \theta_2$).

Based on the geometry of **Figure 8** and the fact that the same target speed is observed by two arrays, the track parameters must satisfy the following conditions:

(1) For track direction,

25 $\theta_1 = \theta - \alpha_1 ; \quad (4)$

Inventor: Shyu
Serial No.

PATENT APPLICATION
Navy Case No. 79,739

5 $\theta_{2r} = \theta - \alpha_2 ;$ (5)

(2) For CPA distance,

$$D_{2r} = |D_1 + L \sin(\delta)| = |D_2 + L \sin(\theta - \beta)| \quad (6)$$

where

$$\delta = \theta - \beta \quad (7)$$

10 (3) For CPA time,

$$t_{02} = t_{01} + L \cos(\delta)/v = t_{01} + L \sin(\theta - \beta)/v, \quad (8)$$

From these constraints, the track parameters relative to the secondary array 96 can be computed from the track parameters relative to the primary array 94. These parameters can be used in Eq. (1) to generate the corresponding delay line curves in the secondary correlogram. 15 This allows the simultaneous integration over delay curves for the primary and secondary correlograms.

The Hough Transformation is a form of the matched spatial filter. See, Slansky, ON THE HOUGH TECHNIQUE FOR CURVE DETECTION, IEEE. Trans. Computer, Vol 27, No. 10, 20 pp. 923-926, 1978. In essence, the Hough Transform hypothesizes a set of prototypes in the image, performs integration along the prototypes, and stores the normalized integration value in the Hough space. The DCHT hypothesizes and integrates pixel values along hypothesized delay curves in a single correlogram. The CHT hypothesizes a reference track relative to the primary array and derives the corresponding delay curve. Given a hypothesized track for the primary array and using the geometric constraints previously discussed, a corresponding delay curve can 25

Inventor: Shyu
Serial No.

PATENT APPLICATION
Navy Case No. 79,739

5 be derived for the secondary correlogram. An integration process is then performed along each of these delay curves; one in the primary correlogram and one in the secondary correlogram. The integrated value from the primary correlogram is then combined with that from the secondary correlogram, and the result is stored in the Hough parameter space.

Normally, targets are detected by searching peaks in the Hough Parameter Space.

10 However, because of the potential problem of correlation misidentification, a different approach called onion-peeling has been developed. See, Brannan et al., DESIGN AND EVALUATION OF TRACK-BEFORE-DETECT PROCESSING FOR ACOUSTIC BROADBAND DATA, Tech. Rpt. No. 1736, Naval Command, Control and Ocean Surveillance Center, RDT&E Div., San Diego, CA, Jan. 1997. Onion peeling is an iterative process whereby 15 the first target is detected by determining the highest peak in the parameter space. Based on the parameter values of the peak, corresponding delay curves are generated in the correlograms and pixel values along these delay curves are replaced with the original mean value of the correlogram. The newly modified correlograms are processed by the CHT and a second target is detected. This process continues either until a fixed number of targets have been detected or the 20 highest remaining peak is below a certain threshold.

There are two versions of Composite Hough Transform: the Additive Composite Hough Transform (ACHT) and the Multiplicative Composite Hough Transform (MCHT). The ACHT can be mathematically represented as.

$$A(\theta_1, \nu, D_1, t_o) = \iint F_1(x, y) \delta(\tau_1(x, y)) dx dy + \iint F_2(x, y) \delta(\tau_2(x, y)) dx dy \quad (9)$$

5 The MCHT is

$$M(\theta_1, v, D_1, t_{o1}) = \iint F_1(x, y) \delta(\tau_1(x, y)) dx dy \times \iint F_2(x, y) \delta(\tau_2(x, y)) dx dy \quad (10)$$

where $A(\theta_1, v, D_1, t_{o1})$ is the value of the ACHT for a source with parameter $(\theta_1, v, D_1, t_{o1})$,

$M(\theta_1, v, D_1, t_{o1})$ is the value of MCHT for a source with parameter $(\theta_1, v, D_1, t_{o1})$, v is target speed, D_1 is CPA horizontal distance from the source to primary array, D_2 is CPA distance from the source 10 to the secondary array; t_{o1} is CPA time of track observed by the primary array, t_{o2} is CPA time of track observed by the secondary array, θ_1 is track direction relative to primary array, θ_2 is track direction relative to secondary array, $\tau_1(x, y)$ is delay curve trace in primary correlogram $(\theta_1, v, D_1, t_{o1})$, $\tau_2(x, y)$ is delay curve trace in secondary correlogram, $F_1(x, y)$ is pixel value of correlogram from primary array, and $F_2(x, y)$ is pixel value of correlogram from secondary array.

15 A straightforward implementation of the CHT can be described as follows:

- (a) hypothesize a track with track parameters values $(\theta_1, v, D_1, t_{o1})$;
- (b) generate the corresponding template delay curve in the primary correlogram based on the delay curve parameter $(\theta_1, v/D_1, t_{o1})$;
- (c) perform integration along the template delay curve in the primary correlogram;
- 20 (d) compute the delay curve parameter $(\theta_1, v/D_2, t_{o2})$ for the secondary array based on the geometric constraints, Eq. (4) through Eq. (8);
- (e) generate the corresponding template delay curve in the secondary correlogram based on the delay curve parameter $(\theta_2, v/D_2, t_{o2})$;
- (f) perform integration along the template delay curve in the secondary correlogram; and
- 25 (g) combine both integrated values and store it in the corresponding Composite Hough Space.

These seven steps are repeated for each individual hypothesized track that is within a set of selected potential tracks. The computation burden and the memory requirement for the CHT 30 are very large. However, relative to the primary array, a reference track and its associated mirror

Inventor: Shyu
Serial No.

PATENT APPLICATION
Navy Case No. 79,739

5 track will generate the same delay curve in the primary correlogram. This relation can be further exploited to reduce the computation burden and memory requirements of CHT. Since a reference track and its associated mirror track will produce the same delay curve in the primary correlogram, the integration values in the primary correlogram for a reference track will be identical to that of its mirror. There is no need to perform the integration process for the delay 10 curve corresponding to a reference track in the primary correlogram. The result of the integration process for the delay curve corresponding to a reference track can be re-used and combined with the integration result of the delay curve corresponding to the associated mirror track in the secondary correlogram. By re-using the result of the integration process for the delay curve corresponding to a reference track in the primary correlogram, 50% of the computational burden 15 for the primary correlogram can be saved. Exploiting another set of geometrical constraints that the associated mirror track has to satisfy can help one to achieve this computational saving:

$$\theta_m = \alpha_1 - \theta_1; \quad (11)$$

$$\theta_{2m} = \alpha_2 - \theta_m; \quad (12)$$

$$D_2 = |D_1 + L (\sin(\delta_m))| = |D_1 + L (\sin(\beta - \theta_m))|; \quad (13)$$

$$\delta_m = \beta - \theta_m; \quad (14)$$

$$t_{02m} = t_{01} + L \cos(\delta_m)/v = t_{01} + L \cos(\beta - \theta_m)/v; \quad (15)$$

where

25 θ_{2m} is the mirror track direction relative to secondary array;
 δ_m is difference between mirror track direction and orientation of the array pair;
 D_{2m} is CA distance from center of secondary track to mirror track;
 t_{02m} is CPA time of mirror track with respect to secondary array, and
 ϕ_{2m} is turn angle at secondary array for mirror track ($\omega_2 - \theta_{2m}$).

5 This set of geometric constraints can be derived using a similar target-sensor geometry shown in **Figure 8**, with the exception that the reference track is replaced by the associated mirror track. The revised implementation for the CHT can be described as follows:

- 10 (a) hypothesize a track with track parameters values (θ_1 , v/D_1 , t_{01});
 (b) generate the corresponding template delay curve in the primary correlogram based on the delay curve parameter (θ_1 , v/D_1 , t_{01});
 (c) perform integration along the template delay curve in the primary correlogram;
- 15 (d) compute the delay curve parameter (θ_2 , v/D_2 , t_{02}) for the secondary array based on the geometric constraints, Eq.(4) through Eq. (8);
 (e) generate the corresponding template delay curve in the secondary correlogram based on the delay curve parameter (θ_2 , v/D_2 , t_{02});
 (f) perform integration along the template delay curve in the secondary correlogram;
- 20 (g) compute the delay curve parameter (θ_{2m} , v/D_{2m} , t_{02m}) for the secondary array based on the geometric constraints, Eq. (11) through Eq. (15);
 (h) generate the corresponding template delay curve in the secondary correlogram based on the delay curve parameter (θ_{2m} , v/D_{2m} , t_{02m});
 (i) perform integration along the template delay curve in the secondary correlogram;
 (j) combine the integrated values and store it in the corresponding Composite Hough Space.

25 In essence, the revised implementation of the CHT simultaneously processes two hypothetical tracks in one processing cycle; one is the reference track and the other is its associated mirror track. On the other hand, the straight forward version processes the reference track and its associated track in two processing cycles. Although the revised implementations of the CHT seems to have more steps than the straightforward version, the saving in computational burden is quite significant.

Depending on the need, the Composite Hough Space can be organized into different configurations. For example, the Composite Hough Space can be organized as two four-dimensional (v , D , θ , t_0) subspaces. One of the subspaces contains all tracks that are positive

5 according to the right-turn rule with respect to the primary array. The other contains all negative tracks. Separating positive and negative tracks this way is equivalent to separating tracks that are mirrors of each other. Therefore, designate the subspace containing the positive tracks as the Reference Track Composite Hough Space (RTCHS) of hypothesized reference tracks for the primary array. And, the subspace of negative tracks is designated the Mirror Track Composite
10 Hough Space (MTCHS). If the RTCHS contains the combined integrated pixel values of the delay curve in the primary correlogram and the corresponding delay curve in the secondary correlogram for a reference track, then the MTCHS will contain the combined integrated pixel values of the delay curve in the primary correlogram and the corresponding delay curve in the secondary correlogram for the associated mirror track. Target detection and track parameter
15 estimation are performed through peak searching and thresholding in both the Composite Hough Spaces. A track is identified as a reference track if the peak is in the RTCHS and a mirror track if the peak is in the MTCHS.

To save memory, the Composite Hough Space can be organized as a combination of the two subspaces; each cell in the combined Composite Hough Space stores the maximum value of
20 the corresponding pixels in the RTCHS and MTCHS. Each cell on the combined Composite Hough Space has an associated flag to indicate whether the cell value comes from the RTCHS or the MTCHS. This configuration does not suppress some information. For example, two different targets, one moving along a reference track and the other along the associated mirror track, will generate a peak in RTCHS and a peak in MTCHS and these two peaks will be combined into one
25 peak. The CHT will then detect one target instead of two. However, the likelihood of two

5 different targets moving exactly together along mirror tracks is negligible. The two-subspace approach is used herein.

Figure 10 shows a data flow diagram of the Composite Hough Transform. Two broadband correlograms, one from primary array (A_1) and one from the secondary array (A_2), the geometric information of sensor arrays, and a set of hypotheses of target track are input to the
10 CHT. The outputs of the CHT are the Reference Track Composite Hough Space and the Mirror Track Composite Hough Space. Each has four dimensions: signal source track direction, CPA tie, CPA horizontal distance, and target speed, where the track direction, CPA time, and the CPA horizontal distance are measured relative to the primary array. Peaks in the Composite Hough Space correspond to potential targets. The location of the peak determines the value of the track
15 parameters.

The following five different simulation cases illustrate:

- (a) basic behavior of the ACHT,
- (b) resolution of the mirror effect by ACHT,
- (c) resolution of the delay curve parameter ambiguity by ACHT,
- (d) behavior of the MCHT, and
- (e) difference in the detection sequence between the ACHT and MCT.

20 Figures 11a through 11c shows two sensor arrays: a primary array (A_1) and a secondary array (A_2) indicated by two circles 104 and 106, respectively, with a short line segment inside each individual circle indicating the orientation of the array 108 and 112, respectively. The thick solid line 114 is the track of the target. Figure 11 b is the primary correlogram with the correlation trace 116 produced by the track 114. Figure 11c is the secondary correlogram. The
25 geometric information for the primary array 104 and the secondary array 106 are shown in Table
solid line 114 is the track of the target. Figure 11 b is the primary correlogram with the correlation trace 116 produced by the track 114. Figure 11c is the secondary correlogram. The
geometric information for the primary array 104 and the secondary array 106 are shown in Table

Inventor: Shyu
Serial No.

PATENT APPLICATION
Navy Case No. 79,739

5 3, and the simulated target's track parameters relative to the primary array 104 are summarized in
Table 4. Note the direction of the simulated target track, 120° , is measured counterclockwise
from east; the CPA range and CPA time are measured with respect to the primary array 104.

TABLE 3

	X-Y coordinates of the array center	Orientation of the array
Primary array	(-1976.13 -729.45) m	-33.61°
Secondary array	(2653.22 773.28) m	151.28°

15 TABLE 4

	Heading	Speed (knt)	CPA range(nmi)	CPA time (sweep)
Track parameter	120°	9	1.0	64

20 **Figures 12a** shows the corresponding Composite Hough Space produced by the ACHT
for $v = 9.0$ kts and $D_1 = 1.0$ nmi; **Figure 12b** shows the MCHTS. The vertical axis is the CPA
time, and the horizontal axis is the track direction; both measured relative to the primary array
104. There are two peaks in the Composite Hough Space, one in the RTCHS and the other in the
25 MTCHS. The peak in the RTCHS is the real peak and has a normalized value of 225. The peak
in the MTCHS is a sidelobe and has a value of 141. Based on this observation, it seems that the
CT is creating a false target. In fact, after the onion-peeling process, both peaks will be removed
as the corresponding correlation traces are removed from the correlograms. When the CHT is

5 performed on these correlograms, there is no peak in the Composite Hough Space with a value above the threshold.

Figure 13a through 13d shows how the CHT can differentiate the reference track from its associated mirror track. Figure 13a shows the sensor-target geometry with both target track 122 and associated mirror track 124. Figure 13b and 13c shows the same correlograms 116 and 118 depicted in Figures 11b and 11c. Figure 13 d shows the secondary correlogram 128, with correlation traces produced by a source traveling along the mirror track. Figure 14a and 14b shows the Composite Hough Space produced by the ACHT, using Figure 13a as the primary correlogram and Figure 13d as the secondary correlogram. Like Figures 12a and 12b, there are two peaks in the Composite Hough Space (Figures 14a and 14b); one in the RTCHS and the other in MTCHS. The peak in the MTCHS is the real peak, while the peak in the RTCHS is a sidelobe. This output of ACHT indicates that the actual track is a mirror track. The difference between Figures 14a through 14b and Figures 12a and 12b clearly indicates that, using two correlograms, Composite Hough Transform can discern a reference track from its mirror track.

20 Figures 15a through 15c shows a scenario with a delay curve parameter ambiguity. The sensor array information is the same as shown in Table 3. There are two tracks 132 and 134, with the track parameters given in Table 1. Figure 15a shows the target-sensor geometry for this scenario. Figure 15b shows the primary correlogram, with only one correlation trace 136; the two tracks have the same delay curve parameters relative to the primary array 132. Figure 15c shows the secondary correlogram, with two correlation traces 138 and 142 produced by the two tracks. Since the signal source speed is higher for the RT1 132, its correlation trace has a higher

5 rate of relative time delay change.

Figures 16a through 16d shows the outputs of two-layer onion-peeling produced by the ACHT. **Figure 16a** and **16b** show the first detected primary correlation trace **144**, and secondary correlation trace **146** and **148**. **Figures 16c** and **16d** show the second detected primary correlation trace **152** and the secondary correlation trace **154** and **156**. To demonstrate that the CHT can detect both correlation traces in the secondary correlogram, the correlation trace in the primary correlogram **144** is used twice. A peeling value of one standard deviation above the mean of the input correlogram was used to paint the detected correlation trace during the onion-peeling process in this case.

Figures 17a and 17b show slices from the corresponding Composite Hough Space.

15 **Figure 17a** and **17b** show the slice of RTCHS and MTCHS produced by the ACHT for a speed of 4.5 kts and D_1 of 1.0 nmi while **Figures 17c** and **17d** shows the slice of RTCHS and MTCHS for a speed of 9.0 kts and D_1 of 2.0 nmi. The presence of the two peaks in the Composite Hough Space indicates that the CHT can distinguish different target tracks with identical delay curve parameters relative to the primary array.

20 The Multiplicative Composite Hough Transform (MCHT) tends to suppress the sidelobes in the Composite Hough Space more than the Additive Composite Hough Transform (ACHT). To demonstrate this refer to **Figure 11a**. Correlograms **Figures 11b** and **11c** are input to both the ACHT and MCHT; corresponding Composite Hough Transform produced by the ACHT as shown in **Figure 18a** through **18d**. **Figures 18a** and **18b** show the output Composite Hough Space produced by the ACHT; **Figures 18c** and **18d** show the output Composite Hough Space

Inventor: Shyu
Serial No.

PATENT APPLICATION
Navy Case No. 79,739

5 produced by the MCHT. The sidelobes produced by the MCHT are lower than those produced by the ACHT. The output signal-to-noise-ratio is defined as

$$SNR_{output} = 20 * \log\left(\frac{Peak - Mean}{Stdv}\right) \quad (16)$$

where

10 Peak is the peak value in Composite Hough Space,
Mean is the average value in the Composite Hough Space, and
Stdv is the standard deviation in the Composite Hough Space.

15 The peak statistic for the Composite Hough Spaces shown in **Figures 181 through 18d** are given in **Table 5**.

TABLE 5

	Peak	Mean	Stdv	SNR
	141	18.04	17.96	16.71
20.	RTCHS produced by ACHT	255	18.02	20.45
	MTCHS produced by MCHT	91	11.32	12.25
25	RTCHS produced by MCHT	255	11.52	15.02
				24.20

As previously stated in regard to Composite Hough Transform, the peaks in the RTCHS are real target peaks and have a higher value, and the peaks in the MTCHS are the sidelobes. The

5 key point in **Table 5** is that the difference of output SNR in RTCHS is about 3 dB while it is 0.4 dB in MTCHS. Thus, the MCHT suppresses the sidelobes more and provides a better peak localization. However, this behavior of MCHT is limited in low SNR. **Figures 19a** through **19f** shows a pair of correlograms with SNR of 0.0 dB. **Figure 19a** shows the primary correlogram, and **Figure 19b** shows the secondary correlogram. **Figure 19c** and **19d** show the result of ACHT, 10 **Figures 19e** and **19f** show the result of MCHT. The output SNR is 22.7 for ACHT and 23.2 for MCHT. These two values are very close, confirming that MCHT and ACHT behave the same way in the low input SNR scenario.

The sequence of target detection of the ACHT can differ from that of the MCHT. **Figure 20** shows an example of such an occurrence. There are two targets (A **158** and B **162**). The 15 intensities of the correlation traces of target A **158** in the primary correlogram are dramatically different from those in the secondary correlation and 20 in the secondary correlogram. On the other hand, for target B **162**, the intensities of the correlation traces are 50 in both primary and secondary correlograms.

Figure 21a through **12f** shows the results of the ACHT while **Figure 22a** through **22f** 20 shows the results of the MCHT. In each case, **Figures 21a** and **22a** shows the results of the first layer of onion-peeling and **Figures 21b** and **22b** shows the results of the second layer onion-peeling. Since the sum of the two correlation traces for target A **158** is 120 while that for the two target B **162** correlation traces is 100, the ACHT first detects the target A **158**, then target B **162**. For the multiplicative Composition Hough Transform, the product of the two target A **158** traces 25 is 2000 while the product of the two target B **162** traces is 2500. Thus, the MCHT will first detect

5 the target B 162 and then will detect the target A 158.

EXPERIMENTAL RESULTS WITH REAL DATA

The real data used here is that shown in **Figures 3a and 3b**. These data were collected from a shallow-water environment with heavy surface ship traffic. The geometry of the two 10 neighboring sensor arrays is listed in **Table 1**. The results of onion-peeling are shown in **Figures 23a and 23b**. The traces are as follows: first detected pair **164**, the second detected pair **166**, the third detected pair **166**, the fourth detected pair **168**, and the fifth detected pair **172**. The reconstructed delay curves closely match the real-data correlation traces, with the exception of the third target **166** where the real-data correlation trace near the end of the observation period does 15 not match the line depicting the third trace **166**. This mismatch is not expected; the CHT assumes that the targets maintain a constant course and the third target **166** had a course change near the end of the observation period.

TABLE 6

Target ID	Heading (deg)	CPA range (nmi)	CPA time (sweep)	Speed (kts)
1	291.1	3.0	57	9.0
2	127.6	12.8	116	8.5
3	209.8	12.8	7	9.0
4	109.6	12.8	118	8.5
5	344.2	8.9	96	7.0

25

Table 6 shows the estimated track parameters for the five targets. The heading of the

5 target is measured clockwise relative to north. **Figure 24** shows the reconstructed target tracks and sensor geometry. The Composite Hough Spaces produced by the ACHT are shown in **Figures 25a through 25g**. The results of MCHT are not dramatically different from the ACHT and thus are not shown here.

The technique described herein, the Composite Hough Transform (CHT), is for
10 multitarget-multisensor tracking and demonstrates the usefulness of CHT with real data sets from a shallow-water environment. Unlike Kalman-filter-based techniques, the CHT does not require an initial guess of the number and state of the targets, but can be applied to multitarget-multisensor tracking as long as the target maintains a constant course and speed.

By explicitly exploiting the geometric properties of multiple arrays, the CHT performs
15 data fusion for multiple sensor arrays, resolves ambiguity due to the mirror effect and the delay curve parameter ambiguity associated with a single array, and allows one to reconstruct the target tracks. This technique enables one to do noncoherent integration from sweep to sweep, from delay-time-cell to delay-time-cell, and from array to array. By combining the energy from data across two spatially separated sensor arrays, the ACHT can theoretically provide an additional 3.0
20 dB processing gain over a single array. This technique can easily be extended to multiple arrays to further increase the processing gain and to facilitate sensor systems configuration and management.

Two versions of the Composite Hough Transform have been taught. The multiplicative
form of CHT suppresses sidelobes more and thus can obtain a sharper peak and improve the
25 accuracy of the parameter estimation. However, in low SNR cases (i.e., noise-limited condition),

5 the performance of MCHT is the same as the ACHT. The CHT does not require a large number of calculations to search over the full space. Using the cued method to greatly reduce the size of search space can reduce this disadvantage.

The foregoing algorithm for an additive composite Hough transform is implemented utilizing the follow flow chart diagrams shown in **Figure as 26 through 32**. The computation 10 process is started **165**, as shown on **Figure 26**, by reading in **167** the values of the following parameters:

- (1) sweep duration
- (2) dimension of correlograms
- (3) file name of primary correlogram
- 15 (4) file name of secondary correlogram
- (5) array configuration file
- (6) the integrating thickness of correlation trace
- (7) the peeling thickness of correlation trace
- 20 (8) maximum time delay of sensor array
- (9) length of processed correlogram segment
- (10) number of onion layers to peel
- (11) searching range of target direction
- (12) searching range target speed
- 25 (13) searching range of CPA distance
- (14) searching range of CPA time
- (15) output file name

Next is read in the coordinates and orientation of primary array from the array configuration file **168**, followed by reading in coordinates and orientation of secondary array from the array configuration file **172** from which the distance and relative orientation between the two arrays is 30 calculated **174**. Then, referring to **figure 27**, memory is allocated **176** for:

- (1) primary correlogram

5 (2) secondary correlogram
(3) Hough space
(4) temporary buffers
(5) display buffers.

10 The mean and standard deviation of individual correlogram is then computed 178, the means are used for peeling the detected correlation traces. Following the assignment of a segment number index =1 is established 182, and answering the inquiry establishing that the number "is the segment number index < its upper limit?" 274, from **Figure 32**, the segment of primary and secondary correlograms is read in 184, followed by establishing an onion layer index = 1 186 and 15 answering the inquiry that the number is greater than the "onion layer index < its upper limit?" 268, **Figure 32**, the speed index = its lower limit 188 is entered after confirming that the speed index is greater than its upper limit 262, **Figure 31**. The speed of the target (V) based on speed index is computed 202.

Following the establishment of the CPA range index + its lower limit 204, shown on 20 **Figure 28**, after ascertaining that the "CPA range index < its upper limit?" 256, **Figure 31**, the target CPA range (R_1) relative to the primary array based on CAP range index is calculated 206, as well as the computation of the ratio V/R_1 208. The target trackdirection index = its lower limit is established 212 and when it is ascertained that the "target track direction index < its upper limit" 252, **Figure 31**, the target direction (θ) and the corresponding mirror track direction (θ_m) 25 relative to the primary array based on target track direction index 214 and the geometric properties of the target track: (1) the intersecting point of the target track and the base line of the primary sensor array; and (2) the slope (m) and y-intercept (b) of the track 216 are computed.

5 Using the geometric constraints 218, Figure 29, compute the following corresponding parameters for the same target relative to the secondary array

- (1) CPA range (R_{2r}) and ratio V/R_{2r})
- (2) CPA time offset (for t_{02r})
- (3) TARGET TRACK DIRECTION (θ_{2R})

10 followed with the performance of the same computation for the corresponding mirror track 222, getting the values of (R_{2m} , V/R_{2m} , t_{02m} offset, θ_{2m}). The CPA time (t_{01}) based on the CPA time index and CPA time offsets are computed 226 after assigning a CPA time (primary array) index = lower limit 224 and verifying that the "CPA range index , its upper limit?" 256, Figure 31. Based

15 on the values of :

- (1) CPA time (t_{01} , t_{02r} , t_{02m})
- (2) CPA range (R_1 , R_2 , R_{2m})
- (3) speed (V)
- (4) track direction (θ , θ_{2r} , θ_{2m})
- (5) max_tau

20 compute the delay curves: one in the primary and two in the secondary correlogram.

Based on the selected integrating width of the correlation trace add up pixels values along the computed delay curves 232, Figure 30 and normalize the pixels value sums by dividing them 25 with the pixels numbers along the delay curves 234. It is noted that for multiplicative composite Hough transform the pixels values will be multiplied instead of added in this stage. The normalized pixels values between hypothesized real track and its corresponding mirror track are compared and the larger one is flagged 236, the normalized pixels values are then stored into the corresponding cell in the Hough space 238, keeping to a minimum the normalized pixels value 30 and the corresponding track parameters 242. A series of indexes are established, such as (1) the

5 CPA.time index = CPA time index +1 is established 244 and if it is determined that the "CPA time index <upper limit?" is not met 246; (2) the target track direction index=target track direction index = 1 248 is established, and it if it is determined that the "target track direction index < its upper limit?" is not met; then the CPA range index = CPA range index + 1 is established and if it is determined that the "CPA range index < its upper limit?" 256 is not met; then a speed index =

10 speed index + 1 is established, and if "speed index < its upper limit?" 262 is not met; then you peel out the strongest delay curve in both correlograms by replacing their pixels values with the mean value of individual correlogram 264, **Figure 32**. Again a series of indexes are established, (1) the onion layer index = onion layer index + 1 266, which is tested to see if the "onion layer index < its upper limit?" is not met; (2) segment number index = segment number index + 1 272

15 is established and tested to see if the "segment number index < its upper limit?" 274, if no; the results are written out and all allocated memory is freed 276; upon completion of which the program stops 278..

Although the invention has been described in relation to an exemplary embodiment thereof, it will be understood by those skilled in the art that still other variations and modifications 20 can be affected in the preferred embodiment without detracting from the scope and spirit of the invention as described in the claims.